

Semirings which are union of principal left k -radicals

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Abstract. Here we study the principal left k -radicals of a semiring with semilattice additive reduct and characterize the semirings which are disjoint union of principal left k -radicals via the transitive closure \xrightarrow{l}^∞ of the relation \xrightarrow{l} on a semiring S , given by for $a, b \in S$, $a \xrightarrow{l} b \Leftrightarrow b^n \in \overline{Sa}$ for some $n \in \mathbb{N}$.

1. Introduction

The notion of principal left k -radical plays a vital role in providing the decomposition of semirings with semilattice additive reduct. The distributive lattice decomposition of semirings is one of the elegant techniques for giving the structure of such semirings, and has been given in [1, 6, 7]. In [7], while giving the decompositions of semirings, the simpler components are found to be left k -Archimedean subsemirings, and that too, via k -radicals of left k -ideals. The notion of principal left k -radicals was introduced in [6] following the ideas of Ćirić and Bogdanović [2], and studied its important characteristics. Also, the very notion of principal left k -radicals induces an equivalence relation λ which was found to be the least distributive lattice congruence on a semiring through which those semirings were characterized which are distributive lattices of λ -simple subsemirings. In terms of principal left k -radical, the semirings in which the principal left k -radicals are the least completely semiprime k -ideals, have been decomposed into λ -simple subsemirings. In this paper, we continue to study the class of semirings with semilattice additive reduct, pick up the notion of principal left k -radicals, and show that a semiring S which is a distributive lattices of left k -Archimedean semirings, can be expressed as a union of principal k -radicals if and only if the relation \xrightarrow{l}^∞ is symmetric on S . During this decomposition we find that the principal left k -radicals becomes the least completely semiprime left k -ideals of S . The preliminaries and prerequisites for this article have been discussed in section 2. In section 3, we study principal left k -radicals $\Lambda(a)$, and show that they are the least completely semiprime left k -ideals of a semiring S containing a , where S is a distributive lattice of left k -Archimedean semirings. Finally, we characterize the semirings which

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